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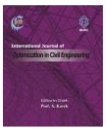

THERMOMECHANICAL TOPOLOGY OPTIMIZATION BASED ON MOVING MORPHABLE COMPONENTS WITH DESIGN-DEPENDENT STEADY-STATE HEAT CONDUCTION

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ABSTRACT

In thermomechanical topology optimization, variations in material distribution influence not only the structural stiffness but also the temperature field and thermal loads induced by constrained thermal expansion. In this study, a moving morphable component (MMC)-based framework is proposed for thermomechanical topology optimization, in which the density-based representation of the SIMP method is replaced with an explicit geometry description using an MMC. In the proposed approach, steady-state heat conduction analysis, equivalent thermomechanical load formulation, and structural mechanical analysis are retained, whereas compliance minimization subject to a volume constraint is adopted as the optimization objective. The elemental density field is derived from the topology description and Heaviside functions, and the sensitivity of the objective function with respect to the geometric parameters of the MMC is evaluated using the chain rule. Numerical examples demonstrate that the proposed method achieves satisfactory thermomechanical performance while producing smoother boundaries and a more explicit geometric representation than the conventional SIMP method.

Keywords: Topology Optimization; Moving Morphable Components (MMC); Thermomechanical Loading; Steady-State Heat Conduction; Compliance Minimization; Explicit Geometric Representation.

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1. INTRODUCTION

Topology optimization is an effective approach for designing lightweight and high-performance structures [1-3]. This enables the determination of the optimal material distribution within a prescribed design domain, subject to a set of constraints and boundary conditions. However, many engineering structures are exposed not only to mechanical loads but also to thermal gradients, elevated temperatures, and environmental temperature variations during their service. Under such conditions, constrained thermal expansion can generate significant thermal stresses and internal forces, thereby substantially affecting the mechanical response of the structure [4, 5]. From the perspective of topology optimization, this issue is even more significant because variations in the material distribution affect not only the structural stiffness but also the temperature field and, consequently, the design-dependent thermomechanical loads [6-8]. Therefore, in thermomechanical topology optimization problems, it is essential to account for the coupled thermal and mechanical behaviors of the structure, as well as the dependence of thermal loads on the evolving design throughout the analysis and optimization processes [9].

To model the coupled effects of heat transfer, thermal expansion, and mechanical response within the optimization process, a significant body of recent research has focused on developing density-based formulations for thermomechanical and thermoelastic problems [10-13]. Zhu et al.[6] investigated the topology optimization of coupled thermomechanical systems by incorporating a temperature constraint and demonstrated that, in the presence of a design-dependent temperature field, both thermal regulation and mechanical response must be simultaneously incorporated into the optimization formulation. Chen et al. [14] extended this line of research to multi-material problems and investigated the influence of temperature-dependent thermoelastic properties on the optimal topology, highlighting that the assumption of constant material properties can be restrictive in many high-temperature applications. Zheng et al. [15] further focused on the role of temperature-dependent material properties in thermoelastic topology optimization and emphasized the importance of updating the material response under a spatially varying temperature field. In a study closely aligned with the objectives of the present work, Ooms et al. [9] proposed a density-based SIMP method for the compliance minimization of structures subjected to thermomechanical loading, in which the temperature field was obtained from a steady-state heat conduction analysis and the equivalent thermomechanical load was evaluated in a design-dependent manner. A key advantage of their work is the use of the adjoint method for sensitivity analysis, which enables the effect of material distribution changes on the temperature field, thermal properties, and thermomechanical loads to be incorporated into the optimization. Extending this line of research, Guo et al. [16] and Ooms et al. [17] further generalized the problem from steady-state to transient thermal conditions and demonstrated that the time-dependent nature of the temperature field and loading can significantly influence the resulting optimal topology and thermomechanical response of a structure. These studies indicate that density-based methods are powerful tools for modeling and solving thermomechanical optimization problems in topology optimization. However, the representation of the final design remains implicit and is dependent on the density field.

Despite the widespread success of density-based methods in solving thermomechanical optimization problems, the final topology is typically obtained in the form of an element-wise

density field; thus, the actual structural boundary is not explicitly defined during the optimization process. Within this framework, extracting the final geometry often requires thresholding, smoothing, or post-processing reconstruction procedures, which may alter the mechanical and thermal responses of the resulting design. Guo et al. [18] identified this limitation as one of the main motivations for developing explicit geometry-based methods, and demonstrated that density-based or nodal-based approaches, although numerically flexible, do not directly embed geometric information into the design variables. This issue becomes even more critical in thermomechanical problems, where intermediate-density regions or pixelated boundaries can complicate the physical interpretation of thermal properties, heat transfer paths, and formation of equivalent thermal loads. Yan et al. [19] also noted in their study on MMC-based thermoelastic topology optimization that density-based methods are typically associated with gray regions and an implicit geometric representation, which can limit the extraction and interpretation of the final design. Consequently, in recent years, increasing attention has been directed toward explicit geometry-based methods, particularly the moving morphable components (MMC) approach, as these methods can directly control the topology through geometric parameters and provide clearer boundaries than density fields [20].

The moving morphable components (MMC) method, one of the most important explicit geometry-based approaches in topology optimization, was originally introduced to directly embed geometric information into the design process. In this framework, the structural topology is not represented by assigning independent densities to each finite element; instead, it is described using a set of geometric components whose positions, sizes, orientations, and shapes are treated as design variables [18]. This shift in design representation enables more explicit control of the structural boundaries and reduces the number of design variables compared with density-based methods. Zhang et al. [21] combined the MMC framework with a replacement material model, enabling its efficient implementation within the finite element method on a fixed mesh, and demonstrated that the advantages of explicit geometric representation can be integrated with the numerical efficiency of mesh-based approaches. Subsequently, Guo et al. [22] enhanced the geometric flexibility of the MMC method by introducing curved skeletal components, thereby enabling the representation of complex topologies. Further developments have shown that the MMC is not limited to classical mechanical problems; for instance, Yu et al. [23] applied this framework to thermo fluid problems and demonstrated its capability in multiphysics applications. In addition, the development of compact and extensible implementations of the MMC method, such as that by Du et al. [24], has made this approach more accessible and practical for numerical implementation in further research. A comprehensive review by Li et al. [20] indicates that MMC has recently become one of the main branches of explicit topology optimization methods, with its advantages primarily lying in explicit geometric representation, reduced dependence on element-wise density fields, and improved interpretability of the final design.

Despite the progress made in applying explicit geometry-based methods to thermoelastic problems, a clear gap still exists between these approaches and design-dependent thermomechanical frameworks based on steady state heat conduction. Yan et al. [19] and Fang et al. [25] demonstrated that the MMC and moving morphable voids (MMV) methods can be effectively employed for the topology optimization of thermoelastic structures, providing sharper and more explicit boundaries than density-based approaches. However, the connection

between these methods and frameworks, in which the temperature field is obtained from a steady-state heat conduction analysis and the equivalent thermomechanical load is constructed in a design-dependent manner, has received less attention. In contrast, Ooms et al. [9] proposed a unified SIMP-based framework for compliance minimization under thermomechanical loading; however, this formulation still relies on a density-based representation. In this context, the present study replaces the density-based representation of this framework with an explicit geometry-based formulation using the moving morphable components (MMC) method, while retaining the thermal analysis, formulation of equivalent thermomechanical loads, and mechanical analysis. The objective of this study is to develop an MMC-based topology optimization approach for thermomechanical problems that achieves a performance comparable to that of the SIMP method while providing smoother boundaries and a more explicit geometric representation.

2. PROBLEM DEFINITION

In this section, a thermomechanical topology optimization framework based on moving morphable components (MMC) is presented. The main idea of the proposed method is to preserve the thermomechanical analysis structure of the reference density-based framework while replacing the SIMP design variables with the geometric parameters of the MMC method. Accordingly, a steady-state heat conduction analysis, formulation of equivalent thermomechanical loads, and structural mechanical analysis were performed in a manner consistent with the reference framework. However, the material distribution is no longer directly controlled by element-wise densities; instead, it is derived from the topology description function of the MMC components combined with the Heaviside projection function. This modification enables the resulting topology to be governed by a reduced set of geometric parameters while providing an explicit representation of the structural boundaries compared with density-based approaches.

2.1 Reference Thermomechanical Framework

In the considered thermomechanical problem, the temperature field within the structure is first obtained by solving the steady-state heat conduction equation as follows: In the discrete form, the heat conduction equation can be expressed as follows [9].

$$K_{th}(\rho)T = Q \tag{1}$$

$$K_{th}(\rho) = \sum_{e=1}^{N_e} k_e\,(\rho_e) K_{th}^{e} \tag{2}$$

The interpolation of the thermal conductivity coefficient is expressed as follows:

$$k_e(\rho_e) = k_{min} + \rho_e^{p_k}(k_0 - k_{min}) \tag{3}$$

where $\mathbf{K}_{th}$denotes the thermal conductivity matrix, $\mathbf{T}$ is the temperature vector, $\mathbf{Q}$ represents the heat flux or thermal load vector, and $\boldsymbol{\rho}$ is the element-wise density field. Since the thermal properties of the material are dependent on the material distribution, variations in element densities lead to changes in the thermal conductivity matrix and, consequently, in the temperature field. The computed temperature field is then used to construct the equivalent thermomechanical load. The mechanical equilibrium equation of the structure is expressed as follows [9]:

$$K(\rho)U = F_m + F_{th} \tag{4}$$

$$K(\rho) = \sum_{e=1}^{N_e} E_e\,(\rho_e) K_0^e \tag{5}$$

In this context, the interpolation of Young's modulus based on the SIMP method is expressed as follows:

$$E_e(\rho_e) = E_{min} + \rho_e^{p_E}(E_0 - E_{min}) \tag{6}$$

$$\boldsymbol{\varepsilon}_{th}^e = \alpha_T(T_e - T_0)\begin{bmatrix}1\\1\\0\end{bmatrix} \tag{7}$$

The thermomechanical load for each element is computed using the following relation [9]:

$$F_{th}^e = \int_{\Omega_e} B_e^T\, D_e \varepsilon_{th}^e\, d\Omega \tag{8}$$

where, in this formulation, the thermal strain of each element is expressed as follows:

$$\boldsymbol{\varepsilon}_{th}^e = \alpha_T(T_e - T_0)\begin{bmatrix}1\\1\\0\end{bmatrix} \tag{9}$$

Finally, the thermomechanical load and the total load applied to the structure are obtained from the following relations:

$$F_{th} = \sum_{e=1}^{N_e} F_{th}^e \tag{10}$$

$$F = F_m + F_{th} \tag{11}$$

where $\mathbf{K}$ denotes the mechanical stiffness matrix, $\mathbf{U}$ is the displacement vector, $\mathbf{F}_m$ represents the external mechanical load vector, and $\mathbf{F}_{th}$ is the equivalent thermomechanical load vector induced by thermal expansion. Within this framework, the thermal load is not

solely a function of the temperature field but also depends on the material distribution; therefore, changes in the structural design simultaneously affect the mechanical stiffness, temperature field, and thermomechanical load. The objective function in this study, consistent with the reference framework, is defined as the minimization of the total structural compliance subject to a volume constraint [9].

$$c(\rho) = F^T U \tag{12}$$

$$V(\rho) = \sum_{e=1}^{N_e} \rho_e v_e \tag{13}$$

$$\frac{\sum_{e=1}^{N_e} \rho_e v_e}{\sum_{e=1}^{N_e} v_e} \leq V_f \tag{14}$$

$$\rho_{min} \leq \rho_e \leq 1, \qquad e = 1,2,\dots,N_e \tag{15}$$

The formulations presented in this section describe only the general framework of the thermomechanical analysis employed in this study. More detailed derivations of the SIMP formulation, interpolation schemes for thermal and mechanical properties, construction of design-dependent thermomechanical loads, and adjoint-based sensitivity analysis can be found in the work by Ooms et al. [9]. Therefore, in this study, the complete set of equations from the reference method is not repeated; instead, only the essential relations required to couple the thermomechanical framework with the MMC-based geometric representation are presented in this section.

2.2 MMC-Based Geometric Representation

In the moving morphable components (MMC) method, the structural topology is represented explicitly by a set of geometric components rather than being implicitly described through element-wise densities. In this approach, each component is characterized by a set of geometric parameters, and the final structural topology is formed by the union of these components. This type of representation enables more explicit control of structural boundaries and provides a clearer geometric interpretation of the final design compared with density-based methods [18, 21].

In the two-dimensional case, a component with constant thickness can be defined using its center coordinates, half-length, half-thickness, and rotation angle. If (x_0, y_0) denotes the center of the component, L is the half-length, t is the half-thickness, and θ represents its orientation angle with respect to the x -axis, then the vector of design variables for the component is defined as follows:

$$X_c = (x_0,\ y_0,\ \theta,\ L,\ t)^T \tag{16}$$

In this formulation, $\mathbf{X}_c$ denotes the vector of geometric parameters of a component. Figure 1 illustrates the geometric parameters of an MMC component with constant thickness.

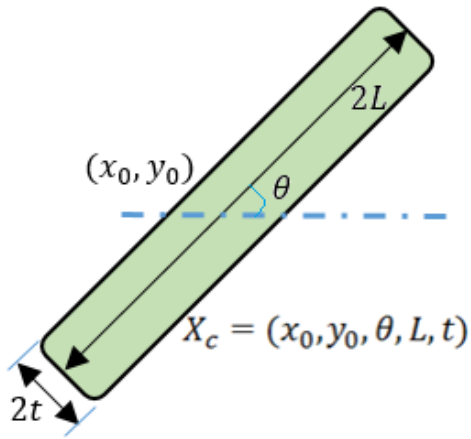


Figure 1: Design parameters of a two-dimensional MMC component with constant thickness [26].

To determine whether a point in the design domain lies inside the component, on its boundary, or outside it, a topology description function TDF is employed. Let D denote the entire design domain, Ω the region occupied by the component, and $\partial\Omega$ its boundary. The position of a point (x, y) with respect to the component is determined by the sign of the function $\Phi(x, y)$ as follows [26]:

$$\Phi(x, y) > 0 \quad \Rightarrow \quad (x, y) \in \Omega \tag{17}$$
$$\Phi(x, y) = 0 \quad \Rightarrow \quad (x, y) \in \partial\Omega \tag{18}$$
$$\Phi(x, y) < 0 \quad \Rightarrow \quad (x, y) \in D\backslash\Omega \tag{19}$$

In the above formulations, $\Phi(x, y)$ denotes the topology description function of the component. To evaluate this function, the coordinates of the current point are first transformed from the global coordinate system to the local coordinate system of the component. This coordinate transformation is expressed as follows:

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} \tag{20}$$

where (x_1, y_1) denotes the coordinates of the considered point in the local coordinate system of the component. For a component with constant thickness, the topology description function TDF is defined as:

$$\Phi(x, y) = 1 - \left(\frac{x_1}{L}\right)^p - \left(\frac{y_1}{t}\right)^p \tag{21}$$

In this formulation, p is a relatively large even integer that controls the shape of the component boundary. Typically, $p = 6$ is used to obtain a component with relatively smooth corners and a well-defined boundary.

The topology of the entire structure is obtained as the union of all components. If the structure consists of n components and Ω_i denotes the region occupied by the $i-th$ component, the overall structural domain is defined by Eq. (22):

$$\Omega^s = \bigcup_{i=1}^{n} \Omega_i \tag{22}$$

This geometric union can also be represented using the TDF of the components. In this case, the topology description function of the entire structure is defined as the maximum of the TDF of all components:

$$\Phi^s(x,y) = \max_{1\le i\le n}\{\Phi_i(x,y)\} \tag{23}$$

Therefore, the material domain, structural boundary, and void region of the entire topology can be defined as:

$$\Phi^s(x,y) > 0 \quad \Rightarrow \quad (x,y) \in \Omega^s \tag{24}$$
$$\Phi^s(x,y) = 0 \quad \Rightarrow \quad (x,y) \in \partial\Omega^s \tag{25}$$
$$\Phi^s(x,y) < 0 \quad \Rightarrow \quad (x,y) \in D\backslash\Omega^s \tag{26}$$

Figure 2 illustrates the concept of forming the overall structural topology as the union of components and the use of the maximum operator to construct the final TDF.

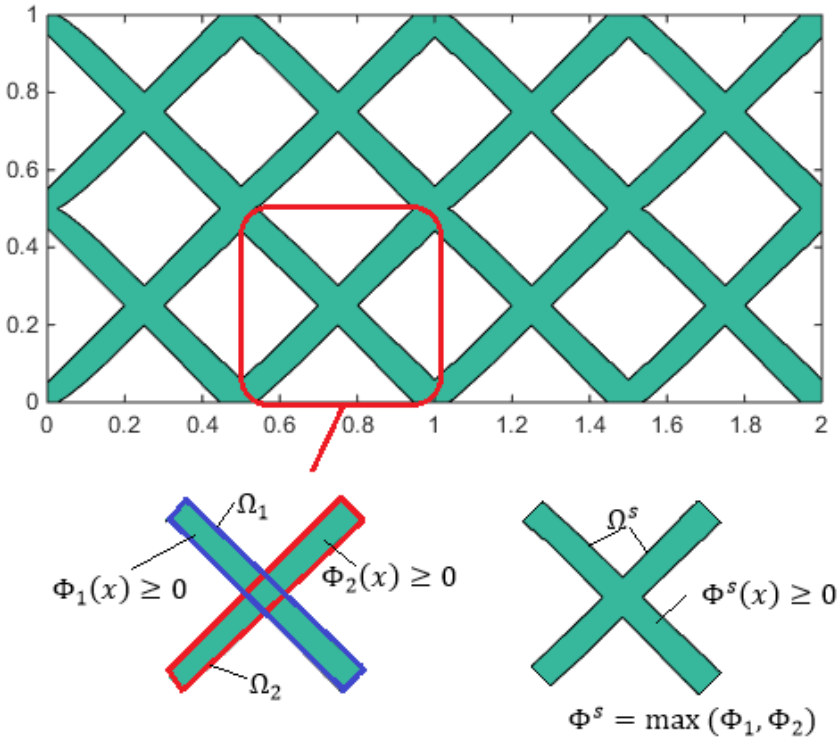


Figure 2: Representation of structural topology using component-wise topology description functions and construction of the global TDF using the maximum operator [26].

To employ the MMC representation in the finite element analysis, the topology description function must be converted into an effective elemental density. In this study, the value of the global TDF is evaluated at the nodes of each element and then transformed into an effective density using a smooth Heaviside function. The density of the $-$e th element can be expressed as the average value of the Heaviside function evaluated at the four nodes of the element [21, 26]:

$$\rho_e = \frac{1}{4}\sum_{q=1}^{4} H_\alpha\left(\Phi^s(\mathbf{x}_{e,q})\right) \tag{27}$$

where $\mathbf{x}_{e,q}$ denotes the coordinates of the q-th node of the e-th element. The smoothed Heaviside function is defined as follows:

$$H_\alpha(\Phi) = \frac{1}{2}[1 + \tanh(\alpha\Phi)] \tag{28}$$

where α is a parameter controlling the sharpness of the transition between the solid and void regions. Therefore, in the proposed method, the element-wise density is no longer treated as an independent design variable but is instead expressed as a function of the geometric parameters of the MMC components:

$$\rho_e = ho_e(\mathrm{X}) \tag{29}$$

This relationship serves as the coupling point between the MMC-based geometric representation and the thermomechanical analysis framework adopted in this study. Accordingly, heat conduction analysis, thermomechanical load generation, and mechanical analysis were performed on a fixed mesh, whereas the material distribution was controlled by geometric components. Consequently, the proposed framework integrates an explicit geometric representation of the MMC with a density-based thermomechanical analysis formulation.

2.3 Coupling of MMC Representation with Thermomechanical Analysis and Sensitivity Analysis

As shown in the previous section, the effective density of each element is a function of the geometric parameters of the MMC component. Therefore, unlike the SIMP approach, in which element densities are treated directly as design variables, the proposed method adopts the geometric parameters of components as the primary design variables. For a compact representation, the global design vector is defined by Eq. (30)[26].

$$\mathrm{X} = \left[\mathrm{X}_{c,1}^T, \mathrm{X}_{c,2}^T, \dots, \mathrm{X}_{c,n}^T\right]^T \tag{30}$$

In this case, the density of each element is a function of the global design vector:

$$\rho_e = \rho_e(\mathrm{X}) \tag{31}$$

The objective function in the thermomechanical framework is first evaluated with respect to the element-wise densities. Therefore, it can be written as:

$$c = c(\rho) \tag{32}$$

Since the densities are functions of the MMC parameters, the final objective function can also be expressed in terms of the geometric design vector:

$$c = c(\mathrm{X}) \tag{33}$$

To evaluate the sensitivity of the objective function with respect to each geometric parameter, the chain rule is employed. If X_j denotes a scalar component of the design vector **X**, the following relation holds:

$$\frac{\partial c}{\partial X_j} = \sum_{e=1}^{N_e} \frac{\partial c}{\partial \rho_e} \frac{\partial \rho_e}{\partial X_j} \tag{34}$$

In the above relation, the term $\partial c / \partial \rho_e$ represents the sensitivity of the thermomechanical objective function with respect to the density of e-th element, which is obtained from the reference thermomechanical analysis framework. The term $\partial \rho_e / \partial X_j$ describes the effect of variations in the MMC geometric parameter on the elemental density.
Based on the definition of the elemental density in Eq. (15), the derivative of the element density with respect to the geometric parameter X_j can be written as follows [21]:

$$\frac{\partial \rho_e}{\partial X_j} = \frac{1}{4} \sum_{q=1}^{4} \frac{\partial H_\alpha}{\partial \Phi^s} \left(\Phi^s \left(\mathbf{x}_{e,q} \right) \right) \frac{\partial \Phi^s \left(\mathbf{x}_{e,q} \right)}{\partial X_j} \tag{35}$$

Since the smoothed Heaviside function is defined in Eq. (16), its derivative with respect to the TDF is given by:

$$\frac{\partial H_\alpha}{\partial \Phi} = \frac{\alpha}{2} [1 - \tanh^2(\alpha \Phi)] \tag{36}$$

If the global TDF of the structure is defined using the maximum operator, then at each node, only the component that yields the maximum TDF value is considered the active component:

$$i^*(\mathbf{x}_{e,q}) = \arg\max_{1 \le i \le n} \{ \Phi_i(\mathbf{x}_{e,q}) \} \tag{37}$$

Therefore, the derivative of the global TDF with respect to the geometric parameter X_j is obtained from the derivative of the TDF of the active component:

$$\frac{\partial \Phi^s(\mathbf{x}_{e,q})}{\partial X_j} = \frac{\partial \Phi_{i^*}(\mathbf{x}_{e,q})}{\partial X_j} \tag{38}$$

However, if the parameter X_j does not belong to the active component, the corresponding derivative becomes zero. By substituting the above relations, the final sensitivity of the objective function with respect to the geometric parameter X_j can be expressed as follows:

$$\frac{\partial c}{\partial X_j} = \sum_{e=1}^{N_e} \frac{\partial c}{\partial \rho_e} \left[\frac{1}{4} \sum_{q=1}^{4} \frac{\alpha}{2} \left(1 - \tanh^2 \left(\alpha \Phi^s(\mathbf{x}_{e,q}) \right) \right) \frac{\partial \Phi^s(\mathbf{x}_{e,q})}{\partial X_j} \right] \tag{39}$$

This relationship indicates that thermomechanical density-based sensitivities can be transformed into sensitivities with respect to the MMC geometric parameters without modifying the underlying thermal and mechanical analyses. Consequently, the reference thermomechanical analysis framework is preserved, whereas the optimization process is performed in terms of geometric design variables.

2.4 Formulation of the Proposed Optimization Problem

In the reference framework, the optimization problem is defined as the minimization of compliance under a volume constraint, while the temperature field and thermomechanical loads are dependent on the material distribution [4]. In the proposed method, this structure is preserved; however, the design variables are changed from element-wise densities to the geometric parameters of MMC. Therefore, the final optimization problem is formulated in terms of the design vector **X**.

The objective function in this study is defined as the minimization of the total structural compliance:

$$\min_{X} \; c(X) = F^T(X)U(X) \tag{40}$$

In this formulation, the total load vector is given as the sum of the external mechanical load and the equivalent thermomechanical load:

$$F(X) = F_m + F_{th}(\rho(X), T) \tag{41}$$

The temperature field is obtained by solving the steady-state heat conduction equation:

$$K_{th}(\rho(X))T = Q \tag{42}$$

After computing the temperature field and constructing the equivalent thermomechanical load, the mechanical response of the structure is obtained by Eq. (43):

$$K(\rho(X))U = F_m + F_{th}(\rho(X), T) \tag{43}$$

The volume constraint in the proposed method is defined based on the effective elemental densities. The structural volume is computed as follows:

$$V(\mathrm{X}) = \sum_{e=1}^{N_e} \rho_e\,(\mathrm{X}) v_e \tag{44}$$

where v_e denotes the volume (or area) of the e-th element. The volume constraint is imposed as follows:

$$\frac{V(\mathrm{X})}{V_0} \leq V_f \tag{45}$$

In the above formulation, V_0 denotes the total design domain volume and V_f represents the prescribed allowable volume fraction. Furthermore, to prevent the components from leaving the admissible design domain and to ensure the stability of the optimization process, lower and upper bounds are imposed on the MMC parameters:

$$\mathrm{X}^{\min} \leq \mathrm{X} \leq \mathrm{X}^{\max} \tag{46}$$

Accordingly, the final MMC-based thermomechanical topology optimization problem can be summarized as follows:

$$\min_{\mathrm{X}}\ c(\mathrm{X}) = \mathrm{F}^T(\mathrm{X})\mathrm{U}(\mathrm{X}) \tag{47}$$

subject to the following constraints:

$$\mathrm{K}_{th}\big(\rho(\mathrm{X})\big)\mathrm{T} = \mathrm{Q} \tag{48}$$

$$\mathrm{K}\big(\rho(\mathrm{X})\big)\mathrm{U} = \mathrm{F}_m + \mathrm{F}_{th}(\rho(\mathrm{X}), \mathrm{T}) \tag{49}$$

$$\frac{V(\mathbf{X})}{V_0} \leq V_f \tag{50}$$

$$\mathrm{X}^{\min} \leq \mathrm{X} \leq \mathrm{X}^{\max} \tag{51}$$

This formulation shows that, in the proposed method, thermal and mechanical analyses are performed in the same manner as in the reference framework; however, the design variables are replaced from element-wise densities to the geometric parameters of the MMC. Consequently, the proposed approach combines the advantages of density-based thermomechanical modeling with the benefits of an explicit geometric representation provided by MMC.

2.5 Solution Procedure and Optimization Algorithm

The solution procedure in the proposed method is based on a combination of the MMC geometric representation and density-based thermomechanical analysis. At the beginning of

the process, a predefined number of morphable components are distributed within the design domain, and their initial geometric parameters are assigned to them. These parameters include the coordinates of the centroid, length, thickness, and rotation angle of each component, which are updated during the optimization process [9, 12].

At each optimization iteration, the topology description function is first evaluated for all components, and the global domain topology function is subsequently obtained through the union of all components. Next, the effective elemental density field was derived using a smoothed Heaviside function. This density field is then used as an input for the thermomechanical analysis, where the steady-state heat conduction problem is first solved to compute the temperature field over the design domain. Based on the obtained temperature field, the equivalent thermomechanical load induced by thermal expansion was constructed and applied together with the external mechanical load in the structural analysis [9].

After solving the mechanical problem, the objective function value, volume constraint, and thermomechanical sensitivities were evaluated. The obtained sensitivities with respect to the elemental densities were then transformed into sensitivities with respect to the MMC geometric parameters using the chain rule. Subsequently, the design variables are updated using a gradient-based optimizer, and the procedure is repeated until the convergence criterion is satisfied. In summary, the proposed algorithm comprises the following steps:

1. Initialization of the MMC geometric parameters for all components.
2. Evaluation of the topology description function for each component.
3. Construction of the global domain topology function and extraction of the effective elemental density field
4. Solution of the steady-state heat conduction problem and computation of the temperature field
5. Construction of the equivalent thermomechanical load based on the temperature field and material distribution.
6. Solution of the mechanical equilibrium problem and computation of nodal displacements
7. Evaluation of the objective function, volume constraint, and thermomechanical sensitivities
8. Transformation of sensitivities from elemental densities to MMC geometric parameters using the chain rule.
9. Update of design variables and check of convergence criterion.

A common stopping criterion is the relative change in the objective function between two successive iterations:

$$\frac{\left|c^{(k)} - c^{(k-1)}\right|}{c^{(k-1)}} \leq \varepsilon_c \tag{52}$$

In the above relation, $c^{(k)}$ denotes the value of the objective function at iteration k, and ε_c represents the convergence tolerance.

3. NUMERICAL EXAMPLES

In this section, the performance of the proposed MMC method in solving the thermomechanical topology optimization problem is investigated and compared with that of the density-based SIMP method. In the considered analysis framework, the temperature field is first obtained by solving the steady-state heat conduction problem. Subsequently, the equivalent thermomechanical load is constructed and applied in the mechanical analysis of the structure. This thermomechanical analysis procedure follows the reference method [4].

The input parameters used in the numerical examples are listed in Table 1. The table includes only the parameters that are directly used in the formulation presented in the previous section or are involved in defining the geometry, discretization, volume constraint, material property interpolation, and the smoothed Heaviside function.

Table 1: Input Parameters of the Numerical Problem

Role in formulation	Unit	value	Symbol
Permissible volume fraction in volume constraints	Without unit	0.4	V_f
Penalty power in elastic modulus interpolation	Without unit	3	p_E
Penalty power in thermal conductivity interpolation	Without unit	3	p_k
Sharpness control parameter of the Heaviside function	Without unit	0.01	α
Reference or initial temperature	°C	0	T_a
Temperature applied in thermal boundary condition	°C	20	T_f
Applied mechanical load	kN	10	f_m
Initial parameters related to the MMC method	ini_val=[0.38 0.04 0.06 0.04 0.7]		

3.1 Comparison of MMC and SIMP Results

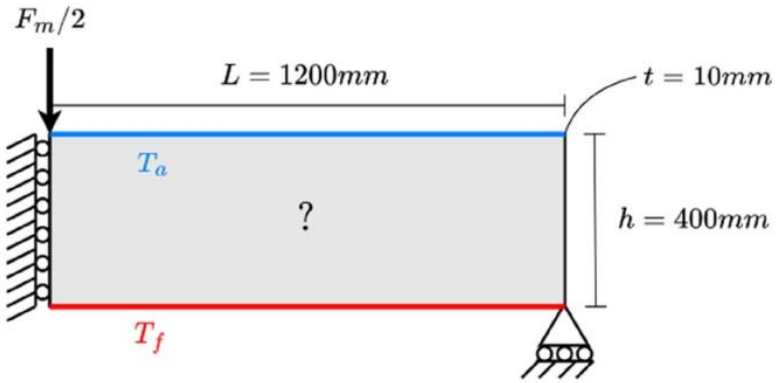
To evaluate the performance of the proposed method, the results obtained from the MMC approach are compared with those of the SIMP method. For a quantitative comparison, the objective function values of both methods are computed. The relative difference in the objective function is defined as follows:

$$\Delta c(\%) = \frac{|c_{MMC} - c_{SIMP}|}{c_{SIMP}} \times 100 \tag{53}$$

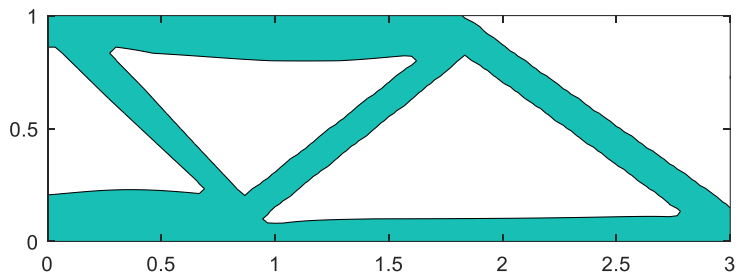
In the above relation, c_{MMC} denotes the objective function value obtained from the proposed method, while c_{SIMP} represents the objective function value obtained from the density-based SIMP method.

Figure 3 illustrates the MBB beam problem under thermomechanical loading, along with the optimized results obtained using both the MMC and SIMP methods. Figure 3(a) presents the geometry of the design domain and the boundary conditions of the problem. A mechanical load of Fm/2 is applied at the upper-left region, while the upper and lower boundaries of the beam are subjected to temperatures T_a and T_f, respectively. Figure 3(b) shows the optimized

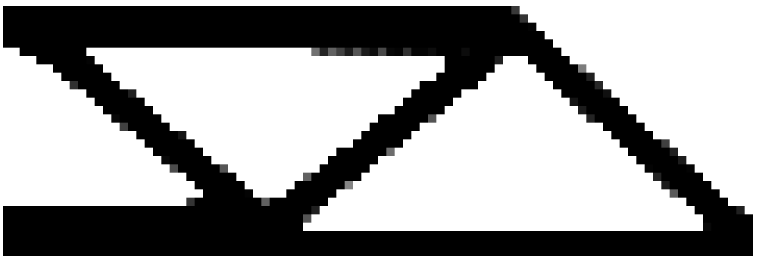
topology obtained using the MMC method. The corresponding compliance value is 29618.28. Figure 3(c) presents the SIMP-based optimized result for the same problem, for which the compliance is 29935.60. The relative difference in compliance between the two methods is 1.0600%. Therefore, the MMC method achieves a numerical performance comparable to SIMP for the MBB beam problem, while producing a final topology with smoother boundaries and a more explicit geometric representation.



(a) Design domain and boundary conditions of the MBB beam



(b) Optimized topology obtained using the MMC method



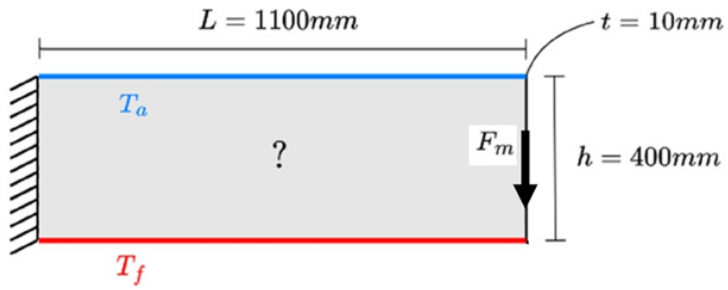
(c) Optimized topology obtained using the SIMP method

Figure 3: MBB beam problem and comparison of optimized topologies obtained using the MMC and SIMP methods

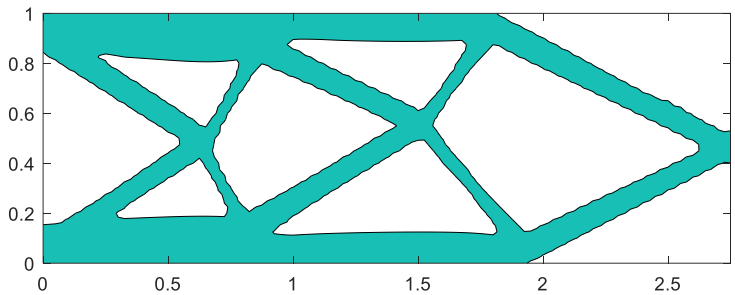
As shown in Figure 3, both MMC and SIMP methods generate similar load-bearing paths for the MBB beam, and the difference in the final compliance values is negligible. However, the geometric representation obtained by the MMC method is smoother and more continuous, whereas the SIMP-based topology appears pixelated and mesh-dependent. This result indicates that the proposed method can improve the geometric quality of the final design

compared to density-based approaches without any noticeable loss in thermomechanical performance.

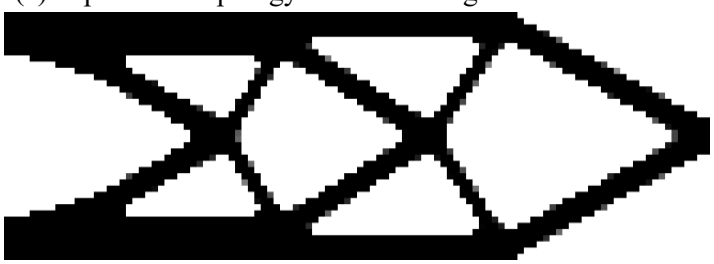
Figure 4 presents the optimized topology results for a cantilever beam subjected to mechanical loading and thermal boundary conditions. The geometry and boundary conditions of the problem have been shown in Figure 4(a); the left side of the beam is fully fixed, and a mechanical load is applied mid-height of the free end. In addition, the upper and lower boundaries of the structure are subjected to temperatures T_a and T_f, respectively. Figure 4(b) illustrates the optimized topology obtained using the MMC method, with a compliance value of 74747.88. In this topology, intersecting diagonal members and continuous load-bearing paths are formed from the load application region toward the fixed support. Finally, corresponding SIMP-based result has been presented in Figure 4(c), for which the compliance is 79611.34. The relative difference in compliance between the two methods is 6.11%. This difference is larger than that observed for the MBB beam case; however, it still demonstrates that the MMC method achieves comparable performance to SIMP while providing smoother and more explicit structural boundaries in the final topology.



(d) Design domain and boundary conditions of the cantilever beam



(e) Optimized topology obtained using the MMC method



(f) Optimized topology obtained using the SIMP method

Figure 4: Comparison of optimized topology of a cantilever beam under thermomechanical loading using MMC and SIMP methods

As shown in Figure 4, for the cantilever beam problem, both the MMC and SIMP methods generate the main load transfer paths from the free end toward the fixed support. The topology obtained by the MMC method exhibits continuous diagonal members and smoother boundaries, and its final structure is represented as a set of identifiable geometric components. In contrast, although the SIMP result also forms similar load-bearing paths, its boundaries are pixelated and mesh-dependent. The compliance values obtained from the MMC and SIMP methods, together with their relative difference, indicate that the proposed method achieves improved geometric quality of the final topology without any significant loss in thermomechanical performance. As shown in Table 2, the MMC method yields lower compliance values than the SIMP method in both numerical examples. The relative reduction in compliance is 1.06% for the MBB beam and 6.11% for the cantilever beam. These results indicate that replacing the density-based representation with an MMC-based geometric representation can not only maintain comparable thermomechanical performance but also lead to an improvement in the structural response.

Table 2: Quantitative comparison of results obtained using MMC and SIMP methods

Problem	c_{MMC}	c_{SIMP}	Relative difference in compliance, Δc	**General Conclusion**
MBB beam	29,618.28	29,935.60	1.06%	Comparable performance to SIMP, accompanied by smoother boundaries in MMC.
Cantilever beam	74,747.88	79,611.34	6.11%	Comparable compliance to SIMP and a more explicit geometric representation in MMC.

3.3 Effect of Boundary Representation on Optimization Results

One of the principal differences between the two methods lies in the representation of the boundary separating the solid and void regions. In the SIMP method, the final boundary is extracted from the elemental density field and therefore typically appears pixelated and mesh-dependent. In contrast, in the MMC framework, the topology is constructed from geometric components and a topology description function (TDF), resulting in boundaries that are smoother, more continuous, and geometrically more interpretable. This distinction is illustrated graphically in Figure 5.

As illustrated in Figure 5, although the SIMP method successfully captures the primary load transfer paths, its resulting boundaries are inherently pixelated and often require additional reconstruction or post-processing before they can be converted into manufacturable geometries. In contrast, owing to its explicit geometric representation, the MMC method generates smoother boundaries that are more readily interpretable from an engineering perspective. This characteristic not only enhances the geometric quality of the final design but may also simplify the preparation of manufacturing models, thereby reducing the time and effort associated with geometry reconstruction. Therefore, the advantages of the MMC approach observed in the present study are not limited to the reduction in compliance; they

also include the generation of topologies with more explicit boundaries and greater engineering applicability.

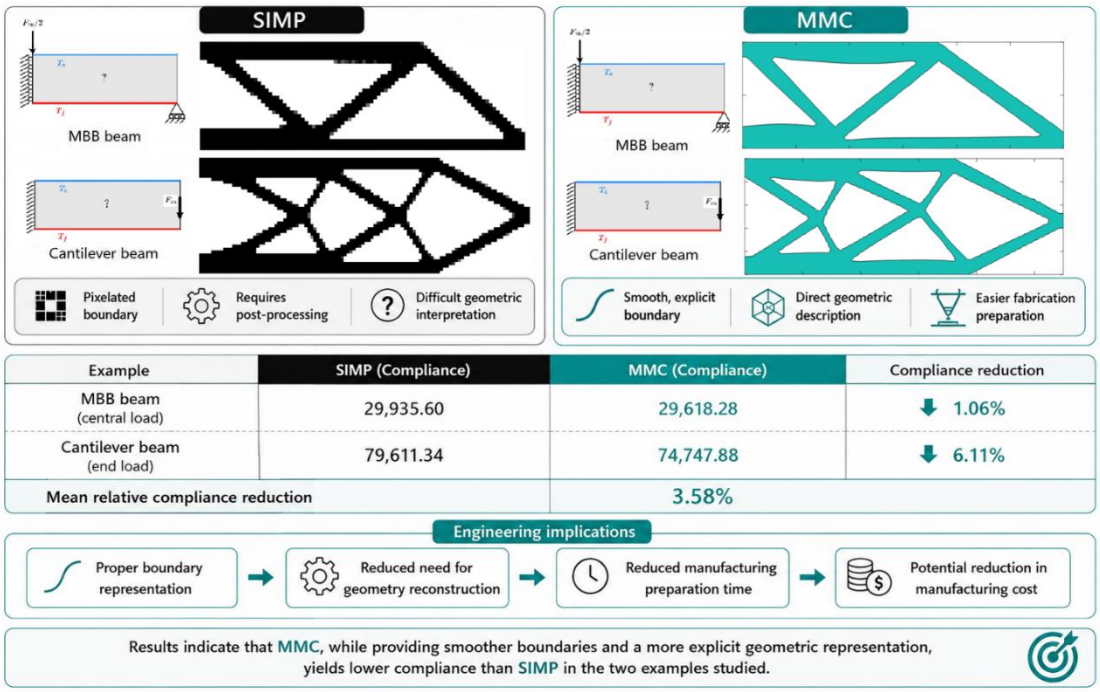


Figure 5: Effect of explicit boundary representation on thermomechanical topology optimization using the MMC method

4. CONCLUSIONS

In this study, a morphable moving components (MMC)-based framework for thermomechanical topology optimization was presented. In the proposed approach, the density-based SIMP representation was replaced by an explicit geometric representation based on MMC, while the steady-state heat conduction analysis, the construction of equivalent thermomechanical loads, and the structural mechanical analysis were preserved. The effective elemental densities were obtained from the topology description function and the Heaviside projection, and the sensitivities of the objective function with respect to the MMC geometric parameters were derived using the chain rule. The numerical results for the MBB beam and the cantilever beam demonstrate that the proposed method achieves a performance comparable to the SIMP method in terms of compliance. For the MBB beam problem, the relative difference in compliance between the two methods is 1.0600%, while for the cantilever beam problem it is 6.11%. These results indicate that the MMC approach can produce reliable thermomechanical responses, while generating final topologies with smoother boundaries and a more explicit geometric representation compared to the density-based method. The advantages of the proposed method are not limited to the reduction of compliance; they also extend to the geometric quality of the final topology. The numerical results demonstrate that, for both benchmark problems considered in this study, the

compliance values obtained using the MMC method are lower than those achieved by the SIMP method. Specifically, the relative reduction in compliance is 1.06% for the MBB beam and 6.11% for the cantilever beam. In addition to this numerical improvement, the topologies generated by the MMC approach exhibit smoother, more continuous, and geometrically more interpretable boundaries. In contrast, due to the density-based nature of the SIMP method, the resulting boundaries are typically pixelated and mesh-dependent. The smoother and more explicit boundary representation provided by the MMC framework reduces the need for geometry reconstruction and post-processing operations, thereby facilitating the transition from topology optimization results to manufacturable designs. Consequently, beyond improving the geometric quality of the final topology, the MMC method has the potential to shorten design-to-manufacturing preparation time and ultimately reduce manufacturing costs. In this study, steady-state thermal analysis and a linear material model were assumed. Accordingly, future work may focus on extending the proposed framework to transient heat conduction, temperature-dependent material properties, stress constraints, three-dimensional problems, and investigating the influence of parameters such as the number of MMC components and thermal load intensity. Overall, the results demonstrate that replacing the density-based representation with an MMC-based geometric representation in thermomechanical topology optimization is feasible and can improve the geometric quality of the final topology while maintaining comparable numerical performance.

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